Simultaneous Formation of Electric and Magnetic Photon States by Electroweak Symmetry Breaking

H. Stumpf

Institute of Theoretical Physics, University of Tübingen, D-72076 Tübingen

Reprint requests to Prof. H. St.; E-mail: herold@tat.physik.uni-tuebingen.de

Z. Naturforsch. **59a,** 185 – 195 (2004); received December 16, 2003

In order to describe the electromagnetic effects (fields) of simultaneously occurring electric and (hypothetical fermionic) magnetic monopoles, Cabibbo and Ferrari introduced in addition to the conventional electric vector potential a magnetic vector potential, thus leading to electric and magnetic photons. A theoretical confirmation (and justification) of this phenomenological ansatz is provided by the manifold of photon states in de Broglie's theory of fusion. Lochak showed that in this theory either electric or magnetic photon states can be derived. To study the possibility of the simultaneous existence of electric and magnetic photon states a modern version of de Broglie's fusion theory is used, which is formulated by means of generalized de Broglie-Bargmann-Wigner equations. It is demonstrated that the corresponding photon equations admit the simultaneous existence of electric and magnetic photon states if the electroweak SU(2)- and CP-symmetry breakings are introduced into these equations. The latter symmetry violations induce violations of the permutation symmetry, which is crucial for the proof of Cabibbo's and Ferrari's hypothesis referred to photons with partonic substructure.

Key words: Electric and Magnetic Photons; Magnetic Monopoles; Relativistic Two-Body Equations; Parastatistics.

1. Introduction

The formal symmetry of Maxwell's equations with respect to the coupling to electric and magnetic sources and the experimentally observed lack of magnetic sources, i. e., the vanishing of the divergence of the magnetic induction, has led to a lot of theoretical speculations in the past century. These speculations can be summarized as the search for magnetic monopoles, [1 – 3]. And recent experimental and theoretical findings gave fresh impetus to this research, [4 – 6].

One widespread argument against the existence of specific magnetic monopoles is offered by duality transformations between electric and magnetic fields. But this sort of solution of the problem is invalidated by serious objections, [2, 5, 7], so the existence of magnetic monopoles is still controversial and imposes an unsolved problem.

Dirac succeeded in deriving magnetic monopoles from solutions of the conventional electromagnetic vector potential, [8, 9]. This leads to a rather sophisticated topological construction giving rise to doubts about the physical realization of such entities. A more natural way seems to be the introduction of a second (magnetic) vector potential by Cabibbo and Ferrari, allowing the description of electromagnetic fields specifically generated by the simultaneous occurence of magnetic and electric monopoles, [10].

In this case it is adequate to distinguish between electric and magnetic photons associated with the respective potential, although both types of photons are only different realizations of the Maxwell field itself.

Support for this way of phenomenological treatment of the monopole problem came from an unexpected quarter: If photons are considered as composites, Lochak showed that in de Broglie's theory of fusion electric and magnetic photons do exist, [11]. In addition Lochak discovered and introduced fermionic magnetic monopoles, [7].

Compared with the widespread treatment of magnetic monopoles as bosons, [12 – 15], this is a new and extraordinary proposal which should be justified by experiment as well as by theory. In particular with respect to the theoretical requirements one should try to extend the standard model by these concepts in order to obtain a theory which corresponds to the theoretical level of modern elementary particle theories and physics and to decide whether new reaction channels of elementary particle reactions can be added to those already known.

In order to reduce the arbitrariness of possible phenomenological modifications of the standard model with respect to such monopoles, a theoretical guideline is needed. In this respect one opportunity of a usefull guideline is provided by the quantum field theoretic generalization of de Broglie's theory of fusion, [16]. It allows to treat the necessary extension of the standard model without suffering from too much arbitrariness.

In de Broglie's original photon theory electric and magnetic photons are mutally exclusive, i.e., either electric or magnetic photons can exist, see [16], Eq.(1.19), Eq.(1.20). On the other hand, according to Cabibbo's and Ferrari's theory both kinds of photons act simultaneously. Clearly in the generalized theory the first step must be the proof that this simultaneous existence and action of both kinds of photons is possible. Thus the first crucial step in analyzing the fermionic monopole problem is the clarification of this problem, for if no medium for the transmission of magnetic monopole effects exists, no magnetic monopole can be detected.

To prove the simultaneous existence of these photons it is sufficient to dicuss in the generalized theory the generalized de Broglie-Bargmann-Wigner equations which describe the structure of a composite photon formed by two constituents (partons or subfermions, respectively). As for the meaning of these equations and their embedding into the complete theory, we refer to previous papers on this topic, [17 – 20], and to a review, [16]. Without repetitions in the following sections the results of these calculations and investigations are used as far as they are important for our discussion.

2. Parafermions by Symmetry Breaking

By means of the field theoretic formalism, wave equations for single composite particles with partonic substructure can be derived. In the most simple case, such composite particles (or quanta) are described by hard core states. The corresponding wave equations are the generalized de Broglie-Bargmann-Wigner (GBBW)-equations which are the object of our treatment.

It is a pecularity of the field theoretic formalism that from the beginning this formalism is not specialized to any definite parton number n. And although in the following we will exclusively deal with the parton number $n = 2$, the general field theoretic formulation is needed in order to be aware of the symmetry properties of the theory.

Hence we start with this field theoretic version of the theory of hard core states which can be expressed by a single covariant functional equation. At this basic level of the theory it is convenient to use only symbolic general coordinate variables I which stand for the four dimensional space-time coordinate x and the algebraic indices Z. Then in this symbolic notation this hard core functional equations reads (using the summation convention)

$$K_{I_1 I}\partial_I|\mathcal{F}\rangle = U_{I_1 I_2 I_3 I_4}[F_{I_2 I} j_I \partial_{I_4}\partial_{I_3} + F_{I_3 I} j_I \partial_{I_2}\partial_{I_4} F_{I_4 I} j_I \partial_{I_3}\partial_{I_2}]|\mathcal{F}\rangle. \quad (1)$$

Definitions of the various quantities which are contained in this symbolic equation will be given below. At first we explain the states $|\mathcal{F}\rangle$. These states are defined by

$$|\mathcal{F}\rangle = \varphi_n(I_1, ..., I_n) j_{I_1} ... j_{I_n}|0\rangle, \quad (2)$$

where φ_n is a formally normal ordered matrix element of the parton dynamics for hard core states, while the set of base vectors $\{j_{I_1}...j_{I_n}|0\rangle\}$ is defined to be a fermionic Fock space with creation operators j_I and their duals ∂_K which have not to be confused with ordinary particle creation and annihilation operators of quantum field theory. They are the elements of the generating functional states and are assumed to satisfy the relations

$$[j_I, \partial_K]_+ = \delta_{IK}; \quad [j_I, j_K]_+ = [\partial_I, \partial_K]_+ = 0; \quad \partial_I|0\rangle = 0\,\forall I. \quad (3)$$

And although these operators are not to be identified with conventional field operators, they reflect an essential property of the parton system.

Assuming the anticommutation relations (3) enforces the wave functions (or matrixelements) φ_n to be completely antisymmetric, and this in turn means: all partons are mutually indistinguishable.

This can be easily verified by considering matrix elements for $n = 2$ of the full theory. In this case the normal ordered matrix elements are identical with the time ordered ones, i. e. $\varphi_2 = \tau_2$, where for fermions the latter are defined by

$$\tau_2 := \langle 0|\Theta(t_1 - t_2)\psi_{Z_1}(\mathbf{r}_1, t_1)\psi_{Z_2}(\mathbf{r}_2, t_2) - \Theta(t_1 - t_2)\psi_{Z_2}(\mathbf{r}_2, t_2)\psi_{Z_1}(\mathbf{r}_1, t_1)|a\rangle. \quad (4)$$

These elements are antisymmetric for $t_1 \neq t_2$ by definition. For equal times, however, antisymmetry can only be achieved if the field operator algebra is exclusively constituted by anticommutation relations, which is the case if the parton fields describe indistinguishable quanta.

So by the relations (3) as well as by the associated field operator algebra the hypothetical fact is expressed that no experiments can be found which enable one to identify different partons (even if partons are assumed to be unobservable).

This situation is changed if symmetry breaking is introduced. Symmetry breaking means that the invariance of the Hamiltonian against a certain symmetry group is violated. In consequence, the degeneracy of the energy eigenvalues of the Hamiltonian with respect to the members of group representations is removed, i.e. the members of a multiplet aquire different energy eigenvalues and can thus be individually identified, i.e. they are no longer indistinguishable.

It is obvious that this fact must find its expression in the properties of the field operator algebra. As the field operators themselves are members of a multiplet of the corresponding group their energetic nondegeneracy makes them distinguishable and thus the anticommutation relations for the case of the unbroken group must be partially replaced by commutation relations for the case of the broken group. I.e., each member of the multiplet has its own fermion state space which does not interact with the fermion state spaces of the other members via the exclusion principle. This necessity is simply illustrated by the fact that in the hydrogen atom the wave function is not antisymmetrized between proton and electron.

In this case according to Green, [21], such a construction with mixed commutators and anticommutators of fermion field operators should therefore be called a parafermi statistics. In this case in the single time limit in (4) antisymmetry is lost.

Without giving further details of this parafermi algebra for field operators, see [22], we directly discuss the effect of this modified algebra on representations. For quantum fields an infinite number of inequivalent representations exist. According to the GNS-construction the choice of a special representation is effected by fixing the vacuum, see [16]. In the case of GBBW-equations, this construction of the representation space is expressed and effected by the appearance of the propagator F in (1). The latter can be used to fix a definite special vacuum. This dependence of (1) on the vacuum can be used to introduce symmetry breaking into the theory via a corresponding modification of the vacuum, i. e. the propagator.

This idea was already pursued by Heisenberg, [23], however, without using the functional calculus and without discussing the consequences for the permutation group. Indeed the sources j_I and ∂_K of the generating functional must reflect the modification of the field operator algebra in their own algebraic properties, i.e. for symmetry breaking we expect a parafermi algebra for the sources too. This in turn to some extend removes the antisymmetry of the matrix elements φ_n in (2), as is to be expected and required.

The preceding discussion can be summarized as follows: *Any symmetry breaking of a dynamical symmetry group induces a symmetry breaking of the corresponding permutation group*.

It would exceed the scope of this paper to develop the parafermi algebras for field operators and for sources in detail. Based on this summary we rather treat the two parton GBBW-equations directly without reference to these algebras in the background.

3. Two Body Equations with CP Violation

First we shortly review the two body GBBW equations without symmetry breaking. These equtions arise from (1) by projection into the two body coordinate space. With $x \in M^4$ and $Z = (i, \kappa, \alpha)$, where κ means superspin-isospin index, $\alpha =$ Dirac spinor index, $i =$ auxiliary field index, these equations read

$$\begin{aligned}&[D^\mu_{Z_1X_1}\partial_\mu(x_1) - m_{Z_1X_1}]\varphi_{X_1Z_2}(x_1,x_2)\\&= 3U_{Z_1X_2X_3X_4}F_{X_4Z_2}(x_1-x_2)\varphi_{X_2X_3}(x_1,x_1),\end{aligned} \tag{5}$$

and in accordance with the use of the generating functional algebra, defined by the relations (3), the state amplitude φ must be antisymmetric.

By interchange of all arguments in (5) one obtains a second set of equations which, however, imposes no further conditions on the wave function φ, if φ is a solution of (5).

In this representation the following definitions are used:

$$D^\mu_{Z_1Z_2} := i\gamma^\mu_{\alpha_1\alpha_2}\delta_{\kappa_1\kappa_2}\delta_{i_1i_2}, \tag{6}$$

$$m_{Z_1Z_2} := m_{i_1}\delta_{\alpha_1\alpha_2}\delta_{\kappa_1\kappa_2}\delta_{i_1i_2}, \tag{7}$$

$$\begin{aligned}&F_{Z_1Z_2}(x_1-x_2) := -i\lambda_{i_1}\delta_{i_1i_2}\gamma^5_{\kappa_1\kappa_2}\\&\cdot[(i\gamma^\mu\partial_\mu(x_1) + m_{i_1})C]_{\alpha_1\alpha_2}\Delta(x_1-x_2,m_{i_1}),\end{aligned} \tag{8}$$

where Δ is the scalar Feynman propagator. The vertex term in (6) is fixed by the defintion

$$U_{Z_1Z_2Z_3Z_4} := \lambda_{i_1} B_{i_2i_3i_4} V^{\kappa_1\kappa_2\kappa_3\kappa_4}_{\alpha_1\alpha_2\alpha_3\alpha_4}, \tag{9}$$

where $B_{i_2i_3i_4}$ indicates the summation over the auxiliary field indices, and where the vertex is given by

$$V^{\kappa_1\kappa_2\kappa_3\kappa_4}_{\alpha_1\alpha_2\alpha_3\alpha_4} := \frac{g}{2}\{[\delta_{\alpha_1\alpha_2}C_{\alpha_3\alpha_4} - \gamma^5_{\alpha_1\alpha_2}(\gamma^5 C)_{\alpha_3\alpha_4}] \cdot \delta_{\kappa_1\kappa_2}[\gamma^5(1-\gamma^0)_{\kappa_3\kappa_4}]\}_{as(2,3,4)}. \tag{10}$$

The parameters λ_i originate from the regularization procedure.

Equations (5) are relativistically invariant quantum mechanical two body equations with nontrivial interaction, selfregularization and probability interpretation. They possess an associated exact single time energy (relativistic Schroedinger) equation and admit exact solutions. More details can be found in [17–20].

For vector bosons the exact solutions read in a general form

$$\varphi_{Z_1Z_2}(x_1,x_2) = T^a_{\kappa_1\kappa_2}\exp[-i\frac{k}{2}(x_1+x_2)] \cdot A^\mu(k)(\chi_\mu)^{i_1i_2}_{\alpha_1\alpha_2}(x_1-x_2|k). \tag{11}$$

The tensors T^a are representations of the superspin-isospin invariance group, see [18], and we expect that after symmetry breaking, modified solutions will replace the solutions (11) for the case of unbroken symmetries.

Although with equations (5) only vector and scalar bosons can be treated, it has to be emphasized that the field theoretic model in the background of these equations is designed to comprise all kinds of interactions. This means that in the field theoretic vacuum all kinds of electroweak and strong symmetry breakings should occur. Thus apart from electroweak isospin invariance violation and electroweak parity violation also electroweak and strong CP-violations have to be taken into account. In particular the latter violations are of special interest with respect to the formation of electric and magnetic photons.

Therefore, in order to concentrate on this formation of electric and magnetic photons we only treat CP-violation in detail, while the other symmetry breakings are only considered to that extend as far as their results are of importance for our intended calculations. In the case of the electroweak isospin symmetry breaking the set of relevant observables is reduced to the electric charge operator Q and to the fermion number operator F, both of which are indispensible for properly discussing CP-violation.

If one excludes states with fermion number 2, which decouple from the effective dynamics, the following set of antisymmetric (Dirac) matrices T^a is a suitable basis for the superspin-isospin description of electroweak vector bosons:

$$T^a = \{T^0, T^1, T^2, T^3\} \equiv \{C, -\gamma^5\gamma^1 C, -i\gamma^5\gamma^2 C, \gamma^5\gamma^3 C\}, \tag{12}$$

while for scalar bosons the set of symmetric Dirac matrices S^a can be used for the description of superspin-isospin properties

$$S^a = \{S^0, S^1, S^2, S^3\} \equiv \{\gamma^0 C, \gamma^5\gamma^0\gamma^1 C, i\gamma^5\gamma^0\gamma^2 C, -\gamma^5\gamma^0\gamma^3 C\}. \tag{13}$$

After symmetry breaking of the isospin group, these properties are expressed by the eigenvalues of Q and F, where Q and F are exclusively defined in superspin-isospin space in accordance with the field theoretic formalism. One obtains, see [16, 18],

$$Q = (G_3 + \frac{1}{2}F) \tag{14}$$

with the isospin generator

$$G_3 := \frac{1}{2}\begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}_{\kappa_1\kappa_2} \tag{15}$$

and the fermion number generator

$$F := \frac{1}{3}\begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{pmatrix}_{\kappa_1\kappa_2}. \tag{16}$$

In the two body system an eigenstate of (14) and (16) is represented by a superspin-isospin tensor Θ which satisfies the conditions

$$Q_{\kappa_1\kappa}\Theta_{\kappa\kappa_2} + \Theta_{\kappa_1\kappa}Q_{\kappa\kappa_2} = q\Theta_{\kappa_1\kappa_2} \tag{17}$$

and

$$F_{\kappa_1\kappa}\Theta_{\kappa\kappa_2} + \Theta_{\kappa_1\kappa}F_{\kappa\kappa_2} = f\Theta_{\kappa_1\kappa_2} \tag{18}$$

simultaneously, owing to the symmetry of Q and F.

Such eigenstates can be formed by linear combinations of elements of the sets (12) or (13), respectively.

This gives the following result with states and corresponding quantum numbers:

$$\begin{aligned}\frac{1}{2}(S^2+S^1)&=\Theta^{1s}(q=0,f=0),\\ \frac{1}{2}(S^2-S^1)&=\Theta^{2s}(q=0,f=0),\\ \frac{1}{2}(S^3+S^0)&=\Theta^{3s}(q=1,f=0),\\ \frac{1}{2}(S^3-S^0)&=\Theta^{4s}(q=-1,f=0),\end{aligned}\tag{19}$$

for symmetric states, and if the *S*-states are replaced by *T*-states one obtains the corresponding antisymmetric eigenstates:

$$\begin{aligned}\frac{1}{2}(T^2+T^1)&=\Theta^{1a}(q=0,f=0),\\ \frac{1}{2}(T^2-T^1)&=\Theta^{2a}(q=0,f=0),\\ \frac{1}{2}(T^3+T^0)&=\Theta^{3a}(q=1,f=0),\\ \frac{1}{2}(T^3-T^0)&=\Theta^{4a}(q=-1,f=0).\end{aligned}\tag{20}$$

Without additional symmetry breaking the states (20) are associated to the quantum numbers of electroweak vector bosons, where the photon and the Z-boson correspond to linear combinations of Θ^{1a} and Θ^{2a} which make via isospin symmetry breaking the Z-boson massive, while the photon remains massless. By the same mechanism the charged bosons aquire masses. But on the elementary level of the parton bound state formation this removal of mass degeneracy will not be further pursued, because it can be shifted to the corresponding effective theory, see [16], which is not the topic of this paper.

The same pattern of charged and neutral bosons is repeated in the case of pseudoscalar and vector mesons, where for pseudoscalar mesons the symmetric states (19) have to be used. Phenomenologically the mesons are bound states of quarks. The identification of these mesons on the parton level rises the question why two-quark states should possess nontrivial projections into the two-parton sector, if quarks themselves are bound states of partons. Indeed one can find a plausible explanation, the discussion of which would likewise exceed the scope of this paper and thus will be treated elsewhere.

Based on these considerations one can proceed to the discussion of CP-violation. The possible modifications of the GBBW-equations are limited by the condition that in any case CPT-invariance must be conserved. For our discussion we adopt the results of a preceding paper on discrete transformations, see [24].

According to Sect. 2, symmetry breaking takes place via the modification of the vacuum. For GBBW-equations the vacuum is represented by the propagator F in (8). Written in full it reads

$$\begin{aligned}F:=&-i\lambda_{i_1}\delta_{i_1i_2}\gamma^5_{\kappa_1\kappa_2}\\ &\cdot\int \mathrm{d}^4p[(\gamma^\mu p_\mu+m_{i_1})C]_{\alpha_1\alpha_2}\\ &\cdot e^{-ip(x_1-x_2)}(p^2-m^2_{i_1})^{-1}(2\pi)^{-4}.\end{aligned}\tag{21}$$

In [24] the PCT-transformation of time ordered matrix elements was derived, which, applied to F, yields the following relation for $x'=-x$

$$\begin{aligned}F^{\kappa_1\kappa_2}_{\alpha_1\alpha_2}(x_1,x_2)_{i_1i_2}=&-\gamma^0_{\kappa_1\kappa_1'}\gamma^5_{\alpha_1\alpha_1'}\gamma^0_{\kappa_2\kappa_2'}\gamma^5_{\alpha_2\alpha_2'}\\ &\cdot F^{\kappa_1'\kappa_2'}_{\alpha_1'\alpha_2'}(x_1',x_2')'_{i_1i_2}.\end{aligned}\tag{22}$$

By direct calculation one can prove the invariance of F, i. e. $F=F'$.

In the same manner, by means of the formulas of [24] the CP-transformation of F is given by

$$F^{\kappa_1\kappa_2}_{\alpha_1\alpha_2}(x_1-x_2)_{i_1i_2}=\tag{23}$$

$$(\gamma^0\gamma^5)_{\kappa_1\kappa_1'}\gamma^0_{\alpha_1\alpha_1'}(\gamma^0\gamma^5)_{\kappa_2\kappa_2'}\gamma^0_{\alpha_1\alpha_1'}F^{\kappa_1'\kappa_2'}_{\alpha_1'\alpha_2'}(x_1'-x_2')'_{i_1i_2}$$

with $x'=(-\mathbf{r},t)$. Again by direct calculation one can prove the invariance of F under this transformation.

From this it follows: if in F a CP-violating term X has to be established, owing to the required CPT-invariance this term must satisfy the condition

$$-\gamma^0_{\kappa_1\kappa_1'}\gamma^5_{\alpha_1\alpha_1'}\gamma^0_{\kappa_2\kappa_2'}\gamma^5_{\alpha_2\alpha_2'}X^{\kappa_1'\kappa_2'}_{\alpha_1'\alpha_2'}=X^{\kappa_1\kappa_2}_{\alpha_1\alpha_2},\tag{24}$$

while the CP-violation must lead to

$$(\gamma^0\gamma^5)_{\kappa_1\kappa_1'}\gamma^0_{\alpha_1\alpha_1'}(\gamma^0\gamma^5)_{\kappa_2\kappa_2'}\gamma^0_{\alpha_2\alpha_2'}X^{\kappa_1'\kappa_2'}_{\alpha_1'\alpha_2'}\neq X^{\kappa_1\kappa_2}_{\alpha_1\alpha_2}.\tag{25}$$

One easily verifies that such a term can be defined by

$$X^{\kappa_1\kappa_2}_{\alpha_1\alpha_2}=(\gamma^5\gamma^0)_{\kappa_1\kappa_2}C_{\alpha_1\alpha_2}.\tag{26}$$

In the phenomenological theory, i. e., in the standard model, the PC-violation arises via the Yukawa coupling of the hypothetical Higgs fields to the fermions, [12, 25]. After the change of the reference point of the Higgs fields, the Yukawa couplings are transformed into the mass matrix of the fermions. Owing to the experimentally and theoretically unknown Yukawa couplings, this mass matrix can only be parametrized but not definitely fixed, and if in this parametrization a special phase parameter does not vanish, then CP-violation takes place.

To transfer this phenomenological procedure to the parton level suggests itself. Indeed the CP-symmetry breaking term (26) can only be incorporated into F by means of a mass correction term. Thus, in accordance with (26), the PC-violating propagator $F(\delta m)$ can be written in the form

$$F(\delta m) = -i\lambda_{i_1}\delta_{i_1 i_2}\gamma^5_{\kappa_1\kappa}(2\pi)^{-4}\int \mathrm{d}^4 p \frac{[(\gamma^\mu p_\mu + m_{i_1})\delta_{\kappa\kappa_2} + \delta m \gamma^0_{\kappa\kappa_2}]_{\alpha_1\alpha} C_{\alpha\alpha_2}}{(p^2 - m^2_{i_1})} e^{-ip(x_1-x_2)} = F_0 + F_1, \quad (27)$$

where F_0 is to be identified with (21) and where δm contains such a phase parameter. From the representation (27) of this modified vacuum expectation value it follows that, owing to the decomposition of γ^0 by $\gamma^0 = \sigma^3_{\Lambda_1\Lambda_2} \otimes \delta_{A_1 A_2}$, the mass correction term acts in a different way on spinors and charge conjugated spinors, causing thus C-violation. As a consequence of this symmetry breaking in the phenomenological treatment, matter can be absolutely discriminated from antimatter and an unambiguous definition of positive charge is possible, [26]. Thus *C*- or *CP*-violation leads to the discrimination of positive from negative charges, even at the parton level, and thus enforces the introduction of parafermions or parapartons, which manifests itself in the violation of the antisymmetry of states.

The introduction of such a mass correction term into the Dirac operator of the propagator is a hypothesis which has to be confirmed by calculating the phenomenological fermion mass matrix of the effective theory of leptons, quarks and gauge bosons with partonic substructure. A first step to derive such an effective mass matrix was done in [27]. But further investigations are needed in order to relate the properties of F to the structure of this matrix.

For the following we accept (27) as a postulate and study the consequences for the formation of electric and magnetic photons.

As the starting point of this investigation we use the integral form of the GBBW-equations (5). It reads

$$\varphi_{Z_1Z_2}(x_1,x_2) = 3\int \mathrm{d}^4 x G_{Z_1X_1}(x_1 - x) \cdot U_{X_1X_2X_3X_4} F_{X_4Z_2}(x - x_2) \cdot \varphi_{X_2X_3}(x,x), \quad (28)$$

and if in (28) the modified propagator (27) is substituted then, according to our discussion in Sect. 2, the CP-invariance violation by (27) induces a violation of the permutation antisymmetry of the wave functions in (28).

It should, however, be noted that the symmetry properties of the vertex U in (28) are not affected, because U is related to the dynamical law of the underlying quantum field which does not depend on the special representation induced by associated special vacuum expectation values.

4. Parafermionic Photon Eigenstates

To solve (28) for the case of CP-symmetry breaking we apply the general ansatz

$$\varphi_{Z_1Z_2}(x_1,x_2) = \exp[-i\frac{k}{2}(x_1 + x_2)]\chi_{Z_1Z_2}(x_1 - x_2). \quad (29)$$

If the Green function in (28) is expressed by its Fourier transform

$$G_{Z_1Z_2}(x_1 - x_2) = \delta_{i_1i_2}\delta_{\kappa_1\kappa_2}(2\pi)^{-4} \cdot \int \mathrm{d}^4 p(\gamma^\mu p_\mu + m_{i_1})_{\alpha_1\alpha_2}(p^2 - m^2_{i_1})^{-1} e^{-ip(x_1-x_2)}, \quad (30)$$

and if (27), (29) and (30) are substituted in (28), then the center of mass motion can be eliminated and an equation for χ only results. Without loss of generality in the latter equation the special coordinate value $(x_1 - x_2) = 0$ can be considered, leading after summation over i_1, i_2 on both sides to a selfconsistent equation for $\hat\chi$, which is defined by summation over i_1, i_2 in $\chi_{Z_1Z_2}(0)$. With $k' := \frac{1}{2}k$ this equation reads

$$\hat\chi^{\kappa_1\kappa_2}_{\alpha_1\alpha_2} =$$

$$g'\int \mathrm{d}^4 p \sum_i \lambda_i f_{i+}[(p_\mu + k'_\mu)\gamma^\mu + m_i]_{\alpha_1\alpha}\delta_{\kappa_1\kappa}$$

$$\cdot \sum_h [v^h_{\alpha\beta}\delta_{\kappa\rho}(v^hC)_{\beta_1\beta_2}\gamma^5_{\rho_1\rho_2} - v^h_{\alpha\beta_1}\delta_{\kappa\rho_1}(v^hC)_{\beta\beta_2}\gamma^5_{\rho\rho_2} - v^h_{\alpha\beta_2}\delta_{\kappa\rho_2}(v^hC)_{\beta_1\beta}\gamma^5_{\rho_1\rho}]$$

$$\cdot \sum_j (-i)\lambda_j\{f_{j-}[(p_\lambda - k'_\lambda)\gamma^\lambda + m_j]_{\beta\delta}C_{\delta\alpha_2}\gamma^5_{\rho\kappa_2} + f_{j-}\delta m C_{\beta\alpha_2}(\gamma^5\gamma^0)_{\rho\kappa_2}\}\hat{\chi}^{\rho_1\rho_2}_{\beta_1\beta_2}, \quad (31)$$

where the definitions

$$f_{i+} := [(p+k')^2 - m_i^2]^{-1}, \quad f_{i-} := [(p-k')^2 - m_i^2]^{-1} \quad (32)$$

were introduced. In addition, in g' all numerical constants are enclosed, being of no relevance for our intended proof.

The reduction of the two-body GBBW equation to an algebraic equation is specific for the two body case and allows an exact solution. GBBW equations for $n > 2$ are much more complicated, see [28].

In the next step $\hat{\chi}$ will be expanded in terms of the complete set of symmetric and antisymmetric Dirac matrices $\{\Gamma_s\}$:

$$\hat{\chi}^{\kappa_1\kappa_2}_{\alpha_1\alpha_2} = \sum_{s=1}^{16} \Gamma^s_{\alpha_1\alpha_2}\chi^{\kappa_1\kappa_2}_s. \quad (33)$$

However, if this expansion is to describe a unique solution of (31) and if this solution is to have a physical meaning, its superspin isospin part must be a unique eigenstate of Q and F in accordance with (17) and (18). This requirement reduces the general ansatz (33) to the special form

$$\hat{\chi}^{\kappa_1\kappa_2}_{\alpha_1\alpha_2} = \sum_{s=1}^{16} c_s\Gamma^s_{\alpha_1\alpha_2}\chi^{\kappa_1\kappa_2}. \quad (34)$$

Furthermore, a closer inspection shows that the omission of the elements C and $(\gamma^5 C)$ can be selfconsistently justified. Thus the most general expansion which is acceptable from a physical point of view reads

$$\hat{\chi}^{\kappa_1\kappa_2}_{\alpha_1\alpha_2} = [A_\mu(\gamma^\mu C)_{\alpha_1\alpha_2} + B_\mu(\gamma^5\gamma^\mu C)_{\alpha_1\alpha_2} + F_{\mu\nu}(\Sigma^{\mu\nu}C)_{\alpha_1\alpha_2}]\chi^{\kappa_1\kappa_2}, \quad (35)$$

and owing to the simultaneous appearance of symmetric and antisymmetric algebra elements this ansatz is a parafermionic one, which reflects the fact that by the mass correction term spinors and charge conjugated spinors can be distinguished.

If this expansion is substituted in (31), after some algebraic rearrangements one obtains

$$\begin{aligned}&[A_\mu(\gamma^\mu C)_{\alpha_1\alpha_2} + B_\mu(\gamma^5\gamma^\mu C)_{\alpha_1\alpha_2} + F_{\mu\nu}(\Sigma^{\mu\nu}C)_{\alpha_1\alpha_2}]\chi^{\kappa_1\kappa_2} = \\ &ig''\int \mathrm{d}^4p \sum_i \lambda_i f_{i+}[(p_\nu + k'_\nu)\gamma^\nu + m_i]_{\alpha_1\alpha}[A_\mu\gamma^\mu_{\alpha\beta} + B_\mu(\gamma^5\gamma^\mu)_{\alpha\beta}](\chi\gamma^5)_{\kappa_1\rho} \\ &\cdot \sum_j \lambda_j\{f_{j-}[(p_\lambda - k'_\lambda)\gamma^\lambda + m_j]_{\beta\delta}C_{\delta\alpha_2}\gamma^5_{\rho\kappa_2} + \delta m f_{j-}C_{\beta\alpha_2}(\gamma^5\gamma^0)_{\rho\kappa_2}\}.\end{aligned} \quad (36)$$

With respect to the superspin-isospin part one observes that for the sets (12) and (13) the relations

$$(T^k\gamma^0) = S^k, \quad (S^k\gamma^0) = T^k, \quad k = 0,1,2,3, \quad (37)$$

or equivalently

$$(\Theta^{ka}\gamma^0) = \Theta^{ks}, \ (\Theta^{ks}\gamma^0) = \Theta^{ka}, \ k = 0,1,2,3 \quad (38)$$

hold. From these relations it follows

$$(\Theta^{ka} + \Theta^{ks})\gamma^0 = (\Theta^{ka} + \Theta^{ks}). \quad (39)$$

Therefore, defining

$$\chi^{\kappa_1\kappa_2}_k = (\Theta^{ka} + \Theta^{ks})^{\kappa_1\kappa_2}, \quad (40)$$

the four possible superspin-isospin states χ_k, $k = 0,1,2,3$ can be eliminated from (36), which yields the system

$$\begin{aligned}&A_\mu(\gamma^\mu C)_{\alpha_1\alpha_2} + B_\mu(\gamma^5\gamma^\mu C)_{\alpha_1\alpha_2} + F_{\mu\nu}(\Sigma^{\mu\nu}C)_{\alpha_1\alpha_2} \\ &= ig''\int \mathrm{d}^4p \sum_i \lambda_i f_{i+}[(p_\nu + k'_\nu)\gamma^\nu + m_i]_{\alpha_1\alpha} \\ &\quad \cdot [A_\mu\gamma^\mu_{\alpha\beta} + B_\mu(\gamma^5\gamma^\mu)_{\alpha\beta}] \\ &\quad \cdot \sum_j \lambda_j\{f_{j-}[(p_\lambda - k'_\lambda)\gamma^\lambda + m_j]_{\beta\delta}C_{\delta\alpha_2} \\ &\qquad + \delta m f_{j-}C_{\beta\alpha_2}\}.\end{aligned} \quad (41)$$

For the further evaluation we introduce the definitions

$$R_{\nu+} := \sum_i \lambda_i f_{i+}(p_\nu + k'_{nu}), \quad S_+ := \sum_i \lambda_i m_i f_{i+} \tag{42}$$

and

$$R_{\lambda-} := \sum_i \lambda_i f_{i-}(p_\lambda - k'_\lambda), \quad S_- := \sum_i \lambda_i m_i f_{i-}. \tag{43}$$

These definitions are substituted in (41), and the right hand side of (41) is expanded in terms of the complete set of symmetric and antisymmetric Dirac matrices.

Comparison of the coefficients on both sides of (41) for each element then yields 16 independent conditions (equations) which must be satisfied by the independent expansion coefficients A_μ, B_μ and $F_{\mu\nu}$.

In particular one obtains for the elements $(\gamma^\mu C)$ the four equations

$$\begin{aligned} A_\mu = ig'' \int \mathrm{d}^4p[R_{\nu+}R_{\lambda-}(\eta^{\nu\rho}\delta_{\lambda\mu} - \eta^{\nu\lambda}\delta_{\rho\mu} \\ + \eta^{\rho\lambda}\delta_{\nu\mu})A_\rho + S_+S_-A_\mu] \\ + ig'' \int \mathrm{d}^4p[i\varepsilon_\mu{}^{\nu\rho\lambda}R_{\nu+}R_{\lambda-}B_\rho + \delta m S_+S_-A_\mu], \end{aligned} \tag{44}$$

and for the elements $(\Sigma^{\mu\nu}C)$ the six equations

$$\begin{aligned} F_{\mu\nu} = ig'' \int \mathrm{d}^4p[-i(S_+R_{\lambda-} - R_{\lambda+}S_-)\delta_{\lambda\mu}A_\nu \\ + \frac{1}{2}\delta m R_+^\lambda S_-\varepsilon_{\mu\nu\rho\lambda}B^\rho] \\ + ig'' \int \mathrm{d}^4p[-i(S_+R_-^\lambda + R_+^\lambda S_-)\varepsilon_{\mu\nu\rho\lambda}B^\rho \\ - i\delta m R_{\nu+}S_-A_\mu], \end{aligned} \tag{45}$$

where all terms on the right hand side are antisymmetrized in μ and ν.

For the elements $(\gamma^5\gamma^\mu C)$ four equations result:

$$\begin{aligned} B_\mu = ig'' \int \mathrm{d}^4p[i\varepsilon_\mu{}^{\nu\rho\lambda}R_{\nu+}R_{\lambda-}A_\rho + S_+S_-B_\mu] \\ + ig'' \int \mathrm{d}^4p[-R_{\nu+}R_{\lambda-}(\eta^{\nu\rho}\delta_{\lambda\mu} - \eta^{\nu\lambda}\delta_{\mu\rho} \\ + \eta^{\rho\lambda}\delta_{\nu\mu})B_\rho + S_+S_-B_\mu], \end{aligned} \tag{46}$$

while the element $(\gamma^5 C)$ yields one equation,

$$\begin{aligned} 0 = ig'' \int \mathrm{d}^4p[-\delta m R_{\nu+}S_-B^\nu \\ + (S_+R_{\lambda-} - R_{\lambda+}S_-)B^\lambda], \end{aligned} \tag{47}$$

and the element C one equation,

$$\begin{aligned} 0 = ig'' \int \mathrm{d}^4p[(S_+R_{\lambda-} + R_{\lambda+}S_-)A^\lambda \\ + \delta m R_{\nu+}S_-A^\nu]. \end{aligned} \tag{48}$$

Owing to symmetry properties, these equations can be considerably simplified. The following relations hold exactly:

$$\int \mathrm{d}^4p(R_{\nu+}R_{\lambda-}\varepsilon^{\rho\nu\mu\lambda}) = 0, \tag{49}$$

$$\int \mathrm{d}^4p(S_+R_{\lambda-} + R_{\lambda+}S_-) = 0, \tag{50}$$

$$\int \mathrm{d}^4p R_{\nu+}R_{\tau-} = -J_1k_\nu k_\tau - J_2\eta_{\nu\tau}, \tag{51}$$

see [18], Eq. (71).

If the Lorentz gauge is assumed to hold for A_μ and B_μ, then (47) and (48) vanish identically, and due to the above relations together with the gauge conditions, (44), (45) and (46) go over into the set of equations

$$A_\mu = ig'' \int \mathrm{d}^4p[R_{\nu+}R_{\lambda-}(\eta^{\nu\rho}\delta_{\lambda\mu} - \eta^{\nu\lambda}\delta_{\rho\mu} + \eta^{\rho\lambda}\delta_{\nu\mu})A_\rho + S_+S_-(1+\delta m)A_\mu], \tag{52}$$

$$F_{\nu\mu} = ig'' \int \mathrm{d}^4p[i(S_+R_{\mu-} - R_{\mu+}S_-)A_\nu + i\delta m R_{\mu+}S_-A_\nu - \frac{1}{2}\delta m R_+^\rho S_-\varepsilon_{\mu\nu\rho\lambda}B^\lambda], \tag{53}$$

$$B_\mu = ig'' \int \mathrm{d}^4p[-R_{\nu+}R_{\lambda-}(\eta^{\nu\rho}\delta_{\lambda\mu} - \eta^{\nu\lambda}\delta_{\mu\rho} + \eta^{\rho\lambda}\delta_{\nu\mu})B_\rho + S_+S_-(1+\delta m)B_\mu], \tag{54}$$

where in (53) the right hand side must be antisymmetrized in μ and ν.

5. Comparison with Phenomenology

To compare the results of our calculation with the phenomenological relations of Cabibbo and Ferrari, equations (52) – (54) have to be evaluated in detail. In this respect use can be made of the calculations in [18]. With (71) and (72) from [18] we have

$$\int \mathrm{d}^4 p S_+ R_{\nu -} = J_\nu^+, \qquad \int \mathrm{d}^4 p S_+ S_- = J_0 \tag{55}$$

with the explicit expressions for the J-terms in (51) and (55)

$$J_0 = -i\pi^2 \sum_{i,j} \lambda_i \lambda_j m_i m_j \int_0^1 \mathrm{d}x \ln|P_{ji}(x)|, \tag{56}$$

$$J_1 = -i\pi^2 \sum_{i,j} \lambda_i \lambda_j \int_0^1 \mathrm{d}x (x - x^2) \ln|P_{ji}(x)|, \tag{57}$$

$$J_2 = \frac{i}{2}\pi^2 \sum_{i,j} \lambda_i \lambda_j \int_0^1 \mathrm{d}x P_{ji} \ln|P_{ji}(x)|. \tag{58}$$

Therefore (52) and (54) go over into

$$A_\mu = ig''[J_1 k^2 + 2J_2 + J_0(1+\delta m)]A_\mu, \tag{59}$$

$$B_\mu = ig''[-J_1 k^2 - 2J_2 + J_0(1+\delta m)]B_\mu, \tag{60}$$

which lead to the eigenvalue conditions

$$\begin{aligned} 1 &= ig''[J_1 k^2 + 2J_2 + J_0(1+\delta m)], \\ 1 &= ig''[-J_1 k^2 - 2J_2 + J_0(1+\delta m)], \end{aligned} \tag{61}$$

and one obtains by addition and substraction of these equations the equivalent set

$$2 = ig'' J_0 (1+\delta m), \tag{62}$$

$$0 = (J_1 k^2 + 2J_2), \tag{63}$$

where in the second equation (63), owing to its homogeneity, the factor ig'' was eliminated by division.

Concerning the further evaluation of (63), the constants J_1 and J_2 are calculated in the limit of equal auxiliary field masses $m_i = m$, $i = 1,2,3$, which corresponds to the use of physical wave functions of the photon. The formulas of this limit for integrals of the kind (56) – (58) were explicitly given in [18], Eq.(89) – (92). Hence J_1 and J_2 can be directly and exactly calculated by means of Mathematica. One obtains

$$\begin{aligned} J_1 = -\frac{i\pi^2}{k^7(-k^2+4m^2)^{-7/2}} &[k(-k^2+4m^2)^{1/2}(k^8 - 26k^6m^2 + 48k^4m^4 + 160k^2m^6 - 480m^8) \\ &- 24m^2(k^8 - 5k^6m^2 + 4k^4m^4 + 40k^2m^6 - 80m^8)\mathrm{ArcTan}(k(-k^2+4m^2)^{-1/2})] \end{aligned} \tag{64}$$

and

$$\begin{aligned} J_2 = -\frac{i\pi^2}{2k^5(-k^2+4m^2)^{-5/2}} &[k(-k^2+4m^2)^{1/2}(k^6 - 12k^4m^2 + 8k^2m^4 + 48m^6) \\ &- 12m^2(k^6 - 4k^4m^2 + 16m^6)\mathrm{ArcTan}(k(-k^2+4m^2)^{-1/2})], \end{aligned} \tag{65}$$

if $k^2 < 4m^2$ holds. Without loss of generality this condition is satisfied for the massless or nearly massless photon states we are interested in. Then one can use the approximation

$$\mathrm{ArcTan}[k(-k^2+4m^2)^{-1/2}] \approx k(-k^2+4m^2)^{-1/2}, \tag{66}$$

and the following explicit expression for the mass eigenvalue equation (63) can be derived:

$$\begin{aligned} 0 &= k^2 + 2J_2 J_1^{-1} \\ &= \frac{2k^2m^2(k^6 + 4k^4m^2 + 40k^2m^4 - 48m^6)}{(k^8 - 6k^6m^2 + 32k^4m^4 + 64k^2m^6 - 160m^8)}. \end{aligned} \tag{67}$$

Obviously this equation possesses the solution $k^2 = 0$, i. e., massless photon states, which is compatible with the integrability condition given above.

In this case one can adopt the value of J_0 for $k^2 = 0$ from [18], Eq. (93), which yields for (62)

$$g'' = -\frac{5m^2}{6\pi^2}(1+\delta m)^{-1}. \tag{68}$$

It should, however, be taken into account that this numerical result is idealized, because the separate eigenvalue equations (61) follow from simplifications which facilitated our calculations, but which need not to be assumed.

In particular, if the vacuum propagator (21) or (27), respectively, differs in its space-time structure from the Green function (30), then in (59) and (60) coupling terms between the A- and B-fields appear, and in consequence only one determinant defines the mass eigenvalue problem. The latter can be solved by assuming $k^2 = 0$ and calculating a corresponding value of the coupling constant g'', as was done in [18].

Hence we can take $k^2 = 0$ for granted, if a more realistic calculation is performed, while in order to achieve maximal transparency, we stick to equations (49) and (50), regardless of the degeneracy in (61).

As far as the field strength tensor is concerned, one can simplify (53) to read

$$F_{\mu\nu} = g''[2i\{J^+_\mu A_\nu\}_{as(\mu\nu)} + \delta m R^\rho_+ S_- \varepsilon_{\mu\nu\rho\lambda} B^\lambda], \tag{69}$$

if the δm correction to the A-term is neglected.

As A_ν as well as B_ν satisfy the homogeneous equations (59), (60), one can renormalize these quantities to give for (69)

$$F_{\mu\nu} = i\{k_\mu A_\nu\}_{as(\mu\nu)} + \varepsilon_{\mu\nu\rho\lambda}\{k^\rho B^\lambda\}. \tag{70}$$

This equation is the Cabibbo-Ferrari definition of electric and magnetic photons in Fourier space. If (70) is represented in coordinate space one obtains

$$\begin{aligned} F_{\mu\nu} &= \{\partial_\mu A_\nu\}_{as(\mu\nu)} + \frac{i}{2}\varepsilon_{\mu\nu\rho\delta}\partial^\rho B^\delta \\ &= \{\partial_\mu A_\nu\}_{as(\mu\nu)} + \{\overline{\partial_\mu B_\nu}\}_{as(\mu\nu)}, \end{aligned} \tag{71}$$

where the factor i is in accordance with the definition of the magnetic vector potential in [7], Eq. (17).

From (71) it follows directly

$$\partial^\nu F_{\mu\nu} = \partial_\mu \partial^\nu A_\nu - \Box A_\mu + \frac{i}{2}\partial^\nu \varepsilon_{\nu\mu\gamma\delta}\partial^\gamma B^\delta, \tag{72}$$

and it can be easily verified that the last term on the right hand side of (72) vanishes identically. Thus, using the eigenvalue equation (59) for A_μ, which must be satisfied for $k^2 = 0$ if (63) holds for this value, one obtains for (72)

$$\partial^\nu F_{\mu\nu} = \partial_\mu \partial^\nu A_\nu - \Box A_\mu = k^2 A_\mu = 0. \tag{73}$$

In addition one gets from (73)

$$\varepsilon^{\mu\lambda\nu\rho}\partial_\lambda F_{\nu\rho} = 0. \tag{74}$$

On the other hand, if we define

$$\bar F_{\alpha\beta} = \frac{i}{2}\varepsilon_{\alpha\beta\gamma\delta}F^{\gamma\delta}, \tag{75}$$

one obtains

$$\begin{aligned} \partial^\beta \bar F_{\alpha\beta} &= \partial^\beta \frac{i}{2}\varepsilon_{\alpha\beta\gamma\delta}(\partial^\gamma A^\delta - \partial^\delta A^\gamma) \\ &\quad + \partial^\beta \frac{i}{2}\varepsilon_{\alpha\beta\gamma\delta}\frac{i}{2}\varepsilon^{\gamma\delta\mu\nu}\partial_\mu B_\nu. \end{aligned} \tag{76}$$

In this case the first term on the right hand side vanishes identically, and evaluation of the second term leads to

$$\partial^\beta \bar F_{\alpha\beta} = -\frac{1}{4}(\partial^\beta \partial_\alpha B_\beta - \Box B_\alpha) = k^2 B_\alpha = 0. \tag{77}$$

Therefore, as a consequence of CP-symmetry breaking, the parton theory of photons just gives the result which was on the phenomenological level postulated by Cabibbo and Ferrari.

As was already emphasized in the introduction, this result is crucial for the theory of magnetic monopoles. Only by the simultaneous existence of electric and magnetic photons magnetic monopoles can exert electromagnetic forces under the simultaneous presence of electric monopoles.

Acknowledgement

I would like to express my thanks to Prof. Alfred Rieckers for a critical reading and discussion of the manuscript.

[1] R. A. Carrigan and W. P. Trower (ed.): Magnetic Monopoles, Plenum, New York 1983.
[2] G. Lehner, Elektromagnetische Feldtheorie, Springer, Heidelberg 1990.
[3] K. A. Milton, G. R. Kalbfleisch, Wei Luo, and L. Gamberg, Intern. J. Mod. Phys. A **17**, 732 (2002).
[4] L. Gamberg, G. R. Kalbfleisch, and K. A. Milton, Found. Phys. **30**, 543 (2000).
[5] G. Lochak, The Symmetry between Electricity and Magnetism and the Problem of the Existence of a Magnetic Monopole, in Advanced Electromagnetism ed. T. W. Barrett, D. M. Grimes, World Scientific, Singapore 1995, p. 1 – 43.
[6] L. I. Urutskoev, V. I. Liksonov, and V. G. Tsinoev, Ann. Fond. L. de Broglie **27**, 701 (2002).
[7] G. Lochak, Intern. J. Theor. Phys. **24**, 1019 (1985).
[8] P. Dirac, Proc. Roy. Soc. A **133**, 60 (1931).
[9] B. Felsager, Geometry, Particles and Fields, Springer, Heidelberg 1998, Chapt. 9.
[10] N. Cabibbo and E. Ferrari, Nuovo Cimento **23**, 1147 (1962).
[11] G. Lochak, Ann. Fond. L. de Broglie **20**, 111 (1995).
[12] Cheng, Ta-Pei and Li, Ling-Fong, Gauge Theory of Elementary Particle Physics, Oxford Univ. Press 1984, Chapt. 15.
[13] K. Huang, Quarks, Leptons and Gauge Fields, World Scientific, Singapore, 1992, Chapt. 5.3.
[14] M. Kaku, Quantum Field Theory, Oxford Univ. Press, 1993, Chapt. 16.
[15] A. Vilenkin and E. P. S. Shellard, Cosmic strings and other Topological Defects, Cambridge Univ. Press, 2000, Chapt. 14.
[16] T. Borne, G. Lochak, and H. Stumpf, Nonperturbative Quantum Field Theory and the Structure of Matter, Kluwer, Dordrecht 2001.
[17] H. Stumpf and T. Borne, Ann. Fond. L. de Broglie **26**, 429 (2001).
[18] H. Stumpf, Z. Naturforsch. **57a**, 723 (2002).
[19] H. Stumpf, Ann. Fond. L. de Broglie **28**, 65 (2003).
[20] H. Stumpf, Generalized de Broglie-Bargmann-Wigner Equations, a Modern Formulation of de Broglie's Fusion Theory, to be published.
[21] H. S. Green, Phys. Rev. **90**, 270 (1953).
[22] Y. Ohnuki and S. Kamefuchi, Quantum Field Theory and Parastatistics, Springer, Heidelberg 1982.
[23] W. Heisenberg, Introduction to the Unified Field Theory of Elementary Particles, Intersc. Publ., London 1966.
[24] H. Stumpf, Z. Naturforsch. **58a**, 481 (2003).
[25] O. Nachtmann, Elementarteilchenphysik, Phaenomene und Konzepte, Vieweg, Wiebaden 1986, Sect. 22.4.
[26] D. Griffiths, Einführung in die Elementarteilchenphysik, Akademie Verlag, Berlin 1996, p. 147.
[27] W. Pfister, Nuovo Cimento **108A**, 1365 (1995).
[28] H. Stumpf, Z. Naturforsch. **58a**, 1 (2003).